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Felipe Aparicio and Javier Estrada *

Abstract

The assumption that stock returns are normally distributed has long been disputed by the data. In this article we test (and clearly reject) the normality assumption using time series of stock returns for each of the four Scandinavian markets during the first half of this decade. More importantly, we fit to the data four alternative specifications, find empirical support for the scaled-*t* distribution, and quantify the magnitude of the error that stems from predicting stock returns by using a Normal distribution.

Keywords
Time series of stock returns. Nonnormality. Forecasting.

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I- INTRODUCTION

The assumption that stock returns are normally distributed is widely used, implicitly or explicitly, in theoretical finance. Investors' preferences can be modeled in a simple way by assuming mean-variance behavior; that is, by assuming that an investor's utility is increasing in the expected return of a portfolio and decreasing in the risk of that portfolio. However, as is well known, mean-variance behavior is consistent with the more general criterium of expected-utility maximization under either one of two conditions, namely, that the investor's utility function is quadratic or that stock returns are normally distributed. Since, as is also well known, a quadratic utility function exhibits some implausible properties,[1] mean variance behavior is usually justified through the assumption of normally-distributed stock returns. Therefore, the widespread use of mean-variance behavior, together with the implausibility of quadratic utility functions, may help to explain the popularity of the normality assumption for stock returns.

From a theoretical point of view, the normality of stock returns is questionable if information does not arrive linearly to the market, or, even if it does, if investors do not react linearly to its arrival; see Peters (1991). Empirical evidence against this assumption, on the other hand, has been mounting since the pioneering articles by Mandelbrot (1963), Fama (1965), and Clark (1973). Mandelbrot (1963) argued that the individual effects making up a price change, though independent, did not have a finite variance; hence, the distribution of stock returns should belong to the family of stable Paretian distributions, which is the only possible family of limiting distributions for sums of *iid* random variables. He directly tested the infinite-variance hypothesis by computing the sample variance of a large number of samples containing the returns of cotton prices, and found that the variances did not converge to any limiting value; rather, they evolved in an erratic fashion, just as would be expected under the infinite-variance hypothesis.

Fama (1965) found that the distribution of daily stock returns was skewed (with a long negative tail) and leptokurtic;[2] he also found that a stable Paretian distribution with a

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[1] A plausible utility function should exhibit decreasing absolute risk aversion, constant relative risk aversion, and increasing risk tolerance. However, the quadratic utility function exhibits increasing absolute risk aversion, increasing relative risk aversion, and decreasing risk tolerance.

[2] The leptokurtosis in stock returns motivated the proliferation of ARCH-type models, which seek to build the information contained in the tails of a distribution of stock returns into time series models. For a literature review, see Bollerslev, Chou, and Kroner (1992).

characteristic exponent less than 2 fitted the data better than a Normal distribution. However, the infinite variance of stable Paretian distributions led many researchers to look for alternatives. Clark (1973) argued in favor of a (finite-variance) subordinated stochastic process and found that a member of this class (the lognormal distribution) fitted data on cotton futures prices better than stable Paretian distributions.

More recently, using weekly data for the period 1928-89, Peters (1991) found that the distribution of the S&P 500 stock returns exhibits negative skewness, fat tails, and a high peak; he also found that the probability of a three-sigma event under the empirical distribution of stock returns is roughly twice as large as the probability that would be expected under a Normal distribution.

A typical explanation for the fat tails is that information does not arrive to the market in a linear fashion; instead, it arrives in infrequent clumps thus forcing the market to react similarly. In other words, since the distribution of information is leptokurtic, so is the distribution of stock returns. However, not everybody subscribes to this theory; see Peters (1991).

The purpose of this study is threefold. First, we test the normality assumption using recent data for the four Scandinavian markets. Second, we attempt to find the specification that best fits the empirical distribution of stock returns in each of the markets under consideration. And, third, we quantify the magnitude of the error that stems from predicting (unconditional) stock returns by using the Normal distribution instead of a more appropriate specification. The rest of the article is organized as follows. In section II, we describe the data and run three tests of normality. In section III, we introduce the statistical distributions to be fitted to the data. In section IV, we present and discuss the results of our estimations. In section V, we assess the error of predicting stock returns by assuming a Normal (instead of a more appropriate) distribution. And, finally, in section VI, we summarize the main findings of our study. An appendix with graphs concludes the article.

II- DATA AND TESTS OF NORMALITY

The sample under consideration consists of the four Scandinavian markets, namely, Denmark (DEN), Finland (FIN), Norway (NOR), and Sweden (SWE). The behavior of each of these markets is summarized by the Financial Times (FT) Actuaries Indices, published daily in the *Financial Times*. In addition, for the purposes of comparison, we analyze the distribution of two additional indices, namely, a European index (EUR) and a World index (WOR).[3] The sample period extends from January 1, 1990, through December 31, 1994; that is, 1,304 daily data points on the first half of the decade. The temporal behavior of the six indices under consideration is shown in part A1 of the appendix.

The series analyzed for each market is the series of returns, where returns are defined as the first difference of the natural logarithm of each index; that is, $R_t = 100*[\ln(I_t) - \ln(I_{t-1})]$, where R_t and I_t are the return and the index in day t, respectively. Table 1 below summarizes some relevant information about the empirical distributions of stock returns under consideration; the statistics reported are the mean, standard deviation, minimum and maximum return during the sample period, coefficients of skewness and kurtosis, and standardized coefficients of skewness and kurtosis.

TABLE 1: Sample Moments of the Distributions of Stock Returns

Market	Mean	SD	Min	Max	Skw	SSkw	Krt	SKrt
DEN	-0.0030	0.8232	-5.8997	4.9312	-0.0936	-1.3803	5.9536	43.8846
FIN	0.0377	1.2440	-5.4757	5.2919	0.2328	3.4316	2.1259	15.6700
NOR	0.0069	1.3307	-8.8584	10.8018	0.3662	5.3988	8.8477	65.2171
SWE	0.0282	1.2504	-6.8453	9.3145	0.5003	7.3755	5.7016	42.0272
EUR	0.0105	0.6908	-6.6946	4.4285	-0.7935	-11.6979	10.8207	79.7604
WOR	-0.0023	0.6535	-4.2796	3.9281	-0.0142	-0.2088	5.5388	40.8272

Sample size=1,304 for all markets. Mean returns, standard deviations (SD), minimum returns (Min) and maximum returns (Max) are all expressed in percentages. Skw=Skewness=m_3/s^3 and Krt=Kurtosis=m_4/s^4-3, where m_i and s are the ith central sample moment and the sample standard deviation of each distribution, respectively; both coefficients are computed with a finite-sample adjustment. SSkw=Standardized skewness and SKrt=Standardized kurtosis.

Preliminary evidence on the normality of each of the distributions under consideration can be gathered from the last four columns of Table 1; that is, by considering the third and fourth central moments of each distribution of returns. Under the assumption of normality, the coefficients of skewness and (excess) kurtosis are asymptotically distributed as $N(0,6/T)$ and $N(0,24/T)$, respectively, where T is the sample size; hence, values of these standardized coefficients (SSkw and SKrt) outside the range [-1.96,1.96] indicate, at the 5% significance level, significant departures from normality.

Table 1 shows that not all the distributions are negatively skewed, as daily data from the U.S. typically shows; this table shows that three distributions (DEN, EUR, WOR) display negative skewness and the other three (FIN, NOR, SWE) display positive skewness. Note,

[3] The European index is an equally-weighted average of the indices of thirteen European countries (Austria, Belgium, Denmark, England, Finland, France, Germany, Italy, Netherlands, Norway, Spain, Sweden, and Switzerland). The World index, on the other hand, is computed on the basis of 2,249 stocks worldwide.

however, that the coefficients of standardized skewness show that the observed skewness is not significant in two (DEN, WOR) out of the six markets under consideration. In addition, the last column of Table 1 shows that *all* six distributions are leptokurtic, thus exhibiting fat tails (and high peaks). The departures from normality detected by the coefficients of standardized skewness and kurtosis can also be seen in the histograms displayed in part A2 of the appendix, where Normal distributions generated by the sample mean and standard deviation of each market are shown together with the observed histograms.

The coefficients of standardized skewness and kurtosis provide strong evidence about departures from normality, but more formal conclusions can be reached through the tests of normality reported below in Table 2. Although the three tests use different information, the results of all three point in the same direction, namely, to the outright rejection of the normality assumption.[4]

TABLE 2: Tests of Normality

	Goodness of Fit			Kolmogorov-Smirnov		Jarque-Bera	
Market	Statistic	df	p-value	Statistic	p-value	Statistic	p-value
DEN	118.647	7	0.0000	0.0716	0.0000	1,927.768	0.0000
FIN	123.383	13	0.0000	0.0612	0.0001	257.335	0.0000
NOR	79.617	8	0.0000	0.0594	0.0002	4,282.456	0.0000
SWE	128.317	9	0.0000	0.0626	0.0001	1,820.680	0.0000
EUR	111.082	6	0.0000	0.0764	0.0000	6,498.599	0.0000
WOR	89.716	9	0.0000	0.0657	0.0001	1,666.898	0.0000

Sample size=1,304 for all markets. The goodness-of-fit test is distributed as a Chi-square with the degrees of freedom (df) indicated above. Asymptotic critical values for the Kolmogorov-Smirnov test, at the 1%, 5%, and 10% significance levels are given, respectively, by 0.045, 0.038, and 0.034. The Jarque-Bera test is asymptotically distributed as a Chi-square with 2 degrees of freedom; critical values at the 1%, 5%, and 10% significance levels are given, respectively, by 9.21, 5.99, and 4.61.

As a result of the strong rejection of the normality assumption applied to the six distributions analyzed, we consider in the next part four alternative distributions that we later fit to the data. We should admit from the outset that we have no underlying financial theory to justify the use of each specification. Our purpose is to fit some distributions that allow for the characteristics of the data discussed above, to determine which of those distributions best fits each market, and to quantify the error made by predicting (unconditional) stock returns by using the Normal distribution instead of the distribution that we select below as the one that best fits each market.

[4] The Jarque-Bera test uses information on the third and fourth moments of a distribution. The goodness-of-fit test divides a distribution in intervals and compares, across intervals, the observed returns with those that would be expected if the underlying distribution were Normal. Finally, the Kolmogorov-Smirnov test computes the maximum distance between an observed cumulative distribution and the Normal cumulative distribution.

III- ALTERNATIVE DISTRIBUTIONS FOR STOCK RETURNS

We consider in this part four distributions of stock returns, which we fit, together with the Normal distribution, to our data in the following part. The results reported and discussed above indicate that the six markets we consider are characterized by generally skewed distributions with fat tails and high peaks; hence, we consider three distributions that allow for these last two characteristics, and one that also allows for skewness.

The Logistic Distribution. This distribution, which is very similar to the Normal but has thicker tails, has been first suggested as appropriate to model stock returns by Smith (1981), and subsequently tested by Gray and French (1990) and Peiró (1994). Its density function can be written as

$$f(x) = \frac{\exp\left(\dfrac{x-\mu}{\alpha}\right)}{\alpha\left[1+\exp\left(\dfrac{x-\mu}{\alpha}\right)\right]^2} \quad , \tag{1}$$

where μ ($-\infty<\mu<\infty$) is a location parameter and α ($\alpha>0$) is a dispersion (or scale) parameter. If R_t follows a logistic distribution, then $E(R_t)=\mu$ and $Var(R_t)=\sigma^2=(\pi^2/3)\alpha^2$.

The Scaled-*t* Distribution. This distribution generalizes the Student's-*t* distribution by allowing for a scale parameter. Praetz (1972), Gray and French (1990), and Peiró (1994) have reported results showing that this distribution fits stock returns better than the Normal distribution. The density function of the scaled-*t* distribution is given by

$$f(x) = \frac{\Gamma\left(\dfrac{v+1}{2}\right)}{\Gamma(v/2)\sqrt{\pi\,(v-2)\sigma^2}} \cdot \left[1+\frac{(x-\mu)^2}{(v-2)\sigma^2}\right]^{-\left(\frac{v+1}{2}\right)} \quad , \tag{2}$$

where $\Gamma(\bullet)$ represents the gamma function, μ ($-\infty<\mu<\infty$) and σ^2 ($\sigma^2>0$) represent a location and a dispersion parameter, respectively, and v ($v>0$) is a parameter that represents the degrees of freedom of the distribution. If R_t follows a scaled-*t* distribution and $v>2$, then $E(R_t)=\mu$ and $Var(R_t)=\sigma^2$.

The Exponential Power Distribution. The family of exponential power distributions, described by Box and Tiao (1973), displays fat tails and high peaks, with the tails shrinking at an exponential rate. Hsu (1982) and Gray and French (1990) have argued that this distribution provides a reasonable good fit to stock return data. The density function of the exponential power distribution is given by

$$f(x) = \frac{\exp\left[-\frac{1}{2}\left|\frac{x-\mu}{\alpha}\right|^{\left(\frac{2}{1+\beta}\right)}\right]}{2^{\left(\frac{3+\beta}{2}\right)}\alpha\,\Gamma\left(\frac{3+\beta}{2}\right)} \,, \tag{3}$$

where μ ($-\infty<\mu<\infty$), α ($\alpha>0$), and β ($-1<\beta\leq 1$) are a location, a dispersion, and a shape parameter, respectively. This last parameter, in particular, measures the kurtosis of the distribution; thus, $\beta<0$ implies a platykurtic distribution, the Normal distribution is obtained when $\beta=0$, and fat tails and a high peak are obtained when $0<\beta\leq 1$, with the thickness of the tails increasing in β.[5] If R_t follows an exponential power distribution, then $E(R_t)=\mu$ and $Var(R_t) = \sigma^2 = 2^{(1+\beta)} \cdot \frac{\Gamma[3(1+\beta)/2]}{\Gamma[(1+\beta)/2]}\alpha^2$.

Mixtures of Two Normal Distributions. An alternative to assuming that stock returns are generated from a single distribution is to assume that they are generated by a mixture of distributions. Press (1967), in particular, argued that stock returns may be generated by the interaction of a continuous diffusion (Brownian motion) process and a discontinuous jump (Poison) process, where the former attempts to capture "standard" changes in stock prices and the second attempts to model large informational shocks. The density function of a mixture of two Normal distributions is given by

$$\begin{aligned} f(x) &= N(\mu_1,\sigma_1^2)\,, \quad \textit{with probability } \lambda \\ &= N(\mu_2,\sigma_2^2)\,, \quad \textit{with probability } (1-\lambda)\,, \end{aligned} \tag{4}$$

where μ_i ($-\infty<\mu_i<\infty$) and σ_i^2 ($\sigma_i^2>0$) are location and a dispersion parameters, respectively. This mixture implies that stock returns are drawn from a Normal distribution with mean μ_1 and standard deviation σ_1 with probability λ, and from a Normal distribution with mean μ_2 and standard deviation σ_2 with probability ($1-\lambda$). If R_t follows such mixture of distributions, then $E(R_t)=\lambda\mu_1+(1-\lambda)\mu_2$ and $Var(R_t)=\lambda\{[\mu_1-E(x)]^2+\sigma_1^2\}+(1-\lambda)\{[\mu_2-E(x)]^2+\sigma_2^2\}$. Of all the specifications we consider, this mixture of two Normal distributions is the only one that allows for skewness in the data.[6]

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[5] For $\beta = 1$, the double exponential distribution is obtained.

[6] The coefficient of skewness (k_3) that follows from the mixture of two Normal distributions is given by

$$k_3 = \frac{\lambda\left[(\mu_1-\mu)^2+3(\mu_1-\mu)\sigma_1^2\right]+(1-\lambda)\left[(\mu_2-\mu)^2+3(\mu_2-\mu)\sigma_2^2\right]}{\left\{\lambda\left[(\mu_1-\mu)^2+\sigma_1^2\right]+(1-\lambda)\left[(\mu_2-\mu)^2+\sigma_2^2\right]\right\}^{3/2}}\,.$$

IV- EMPIRICAL RESULTS

We report in Table 3 below the (maximum likelihood) estimations that result from fitting the theoretical distributions described in the previous part to the observed series of stock returns of the six markets under consideration.

TABLE 3: Parameter Estimates

		DEN	FIN	NOR	SWE	EUR	WOR
N:	μ:	-0.00301	0.03770	0.00689	0.02819	0.01052	-0.00233
	σ:	0.82253	1.24302	1.32967	1.24940	0.69032	0.65305
L:	μ:	-0.00153	0.01425	-0.00854	0.01305	0.02457	-0.00399
	α:	0.41885	0.66172	0.67926	0.64014	0.34677	0.33609
S-t:	μ:	-0.00072	0.00629	-0.01467	0.00929	0.03002	-0.00481
	σ:	0.86401	1.28016	1.30499	1.27510	0.69273	0.66291
	v:	3.34660	4.26650	4.25530	3.69760	3.63030	3.83590
EP:	μ:	-0.00001	-0.00000	-0.00000	0.00000	0.02780	-0.00319
	α:	0.28459	0.51754	0.52770	0.43700	0.26122	0.27211
	β:	0.99999	0.86572	0.88132	1.00000	0.91345	0.84280
MN:	μ_1:	-0.01645	-0.05409	0.00135	-0.00117	-0.21775	0.01230
	σ_1:	1.45330	0.82717	1.06820	0.91374	1.51840	1.20920
	μ_2:	0.00578	0.23524	0.10965	0.22347	0.03788	-0.00536
	σ_2:	0.54173	1.82780	3.66050	2.55450	0.50032	0.46024
	λ:	0.21064	0.68276	0.94890	0.87338	0.10703	0.17163
	k_3:	-0.00157	0.16450	0.03141	0.10426	-0.14151	0.01247

N=Normal. L=Logistic. S-t=Scaled-t. EP=Exponential Power. MN=Mixture of two Normal distributions. The coefficient of skewness (k_3) follows from the expression in footnote 6.

At least two things are worth noting from the previous table. First, recall that the Normal distribution and the t-distribution tend to converge as the degrees of freedom of the latter increase. However, Table 3 shows that the estimated degrees of freedom of the scaled-t distributions are very low in all markets (between 3 and 4.5), thus indicating that these empirical distributions are significantly different from the Normal. Second, recall that the parameter β of the exponential power distribution is a measure of its kurtosis, that for β=0 the Normal distribution is obtained, and that this coefficient is increasing in the thickness of the tails (with an upper bound at β=1). Table 3 shows that β is larger than .8 in all markets and larger than .9 in three markets (DEN, SWE, EUR). This provides additional evidence of departures from normality, and, in particular, of the thickness of the tails of the empirical distributions under consideration.

In order to compare the relative fit of the theoretical distributions we consider and the empirical distributions of stock returns, we performed goodness-of-fit tests.[7] To that

[7] We do not use the likelihood-ratio test for the obvious reason that not all these distributions are nested within each other; hence, their log-likelihood functions are not comparable.

purpose, we divide the range of returns into 20 equal, non-overlapping intervals contained in the range [-10%,10%]. The results of these tests are shown below in Table 4.

TABLE 4: Goodness-of-fit Tests

	N	p-value	L	p-value	S-t	p-value	EP	p-value	MN	p-value
DEN	1.3e6	0.0000	233.2	0.0000	14.7	0.5467	29.4	0.0214	30.7	0.0061
FIN	577.2	0.0000	43.6	0.0000	16.8	0.3987	19.7	0.2340	16.6	0.2781
NOR	22.2e9	0.0000	749.5	0.0004	21.5	0.1601	111.1	0.0000	19.1	0.1612
SWE	2.2e9	0.0000	1,351.9	0.0000	20.7	0.1903	94.8	0.0000	40.7	0.0002
EUR	4.9e14	0.0000	28,777.7	0.0000	11.8	0.7576	1,195.6	0.0000	121.7	0.0000
WOR	1.7e6	0.0000	189.2	0.0000	6.4	0.9832	40.5	0.0007	18.2	0.1978

N=Normal. L=Logistic. S-t=Scaled-t. EP=Exponential Power. MN=Mixture of two Normal distributions. The goodness of fit test follows a Chi-square distribution with p-k-1 degrees of freedom, where p is the number of intervals and k is the number of parameters estimated for each distribution.

The results above show that, as expected, the Normal distribution provides the worst fit among all the specifications considered; it is clearly rejected in all markets. The scaled-t distribution, on the other hand, cannot be rejected in any market at any reasonable significance level. Both the logistic distribution and the exponential power distribution provide a very poor fit, thus being rejected in all markets (with the exception of the exponential power distribution which cannot be rejected in Finland) . Finally, there is partial support for the mixture of two Normal distributions; this specification cannot be rejected at any reasonable significance level in three out of the six markets considered. The overall support we find for the scaled-t distribution confirms results reported by Peiró (1994) for different markets and sample periods. It is interesting to note that, although the data shows that the distribution of stock returns in some markets is skewed, the distribution that provides the overall best fit is the symmetric scaled-t.[8]

V- UNCONDITIONAL FORECASTS OF STOCK RETURNS

The tests of normality reported in part II establish that the distributions of stock returns of the six markets we consider exhibit significant departures from normality. In addition, the goodness-of-fit tests reported in the previous part establish that a scaled-t distribution exhibits the best fit in all markets. In this part, we attempt to assess the magnitude of the error that can be made by predicting (unconditional) stock returns under

[8] We also run the goodness-of-fit test dividing the range of returns into 30 equal, non-overlapping intervals contained in the range [-15%, 15%]; the results obtained were very similar to those displayed in Table 4.

the assumption of a Normal distribution, thus ignoring the information provided by the scaled-*t* distribution.

In order to assess this error, we first estimate the (unconditional) probability of obtaining returns in a given interval using the parameters previously estimated (and reported in Table 3) for the Normal distribution; we subsequently repeat this process for the twelve intervals we consider. Then we estimate the same probability using the parameters previously estimated (and reported in Table 3) for the scaled-*t* distribution for the same twelve intervals. We finally compare, one by one, the probability of obtaining returns in each interval. The results of our estimations are reported below in Table 5.

TABLE 5: Unconditional Forecasts of Stock Returns

		$[\bar{x}, \bar{x}+s]$	$[\bar{x}+s, \bar{x}+2s]$	$[\bar{x}+2s, \bar{x}+3s]$	$[\bar{x}+3s, \bar{x}+4s]$	$[\bar{x}+4s, \bar{x}+5s]$	$[\bar{x}+5s, \bar{x}+6s]$
DEN:	N:	0.34154	0.13579	0.02132	0.00131	0.00003	2.80e-7
	S-*t*:	0.39060	0.08586	0.01701	0.00471	0.00170	0.00074
FIN:	N:	0.34154	0.13580	0.02133	0.00131	0.00003	2.80e-7
	S-*t*:	0.36770	0.09438	0.01836	0.00448	0.00140	0.00053
NOR:	N:	0.34153	0.13580	0.02133	0.00131	0.00003	2.80e-7
	S-*t*:	0.37964	0.08963	0.01630	0.00385	0.00118	0.00044
SWE:	N:	0.34154	0.13580	0.02132	0.00131	0.00003	2.80e-7
	S-*t*:	0.38117	0.08678	0.01674	0.00437	0.00149	0.00061
EUR:	N:	0.34152	0.13582	0.02134	0.00131	0.00003	2.81e-7
	S-*t*:	0.40136	0.09028	0.01670	0.00436	0.00148	0.00061
WOR:	N:	0.34150	0.13580	0.02133	0.00131	0.00003	2.81e-7
	S-*t*:	0.38359	0.09013	0.01721	0.00438	0.00145	0.00058

N=Normal. S-*t*=Scaled-*t*. Each number shows the probability of obtaining a return in the specified interval under the specified distribution. Each distribution is centered around its sample mean ($\bar{x}$), and the length of each interval is equal to each distribution's sample standard deviation (s).

Table 5 shows that the probability of obtaining returns in any given interval is very different depending on whether a Normal or a scaled-*t* distribution is assumed as the underlying distribution. Recall that leptokurtic distributions have a high peak, thus exhibiting clustering of observations around the mean. Accordingly, Table 5 shows that the probability of obtaining returns one standard deviation around the mean is higher under the scaled-*t* than under the Normal distribution in all markets. Furthermore, note that the opposite is the case in the intervals $[\bar{x}+s, \bar{x}+2s]$ and $[\bar{x}+2s, \bar{x}+3s]$; Table 5 shows that the probability of obtaining returns in both intervals is higher under the Normal distribution than under the scaled-*t* distribution in all markets.

Note, however, that the situation reverses again for the interval $[\bar{x}+3s, \bar{x}+4s]$ and all intervals beyond. In other words, the probability of obtaining returns in any of these intervals is higher under the scaled-*t* than under the Normal distribution in all markets.

Furthermore, note that the difference between the probability predicted under each distribution increases dramatically as we move away from the mean. To illustrate, the probability of obtaining a return between three and four standard deviations is on average over three times higher under the scaled-*t* distribution; the probability of obtaining a return between four and five standard deviations is on average almost fifty times higher under the scaled-*t* distribution; and the probability of obtaining a return between five and six standard deviations is on average *over two thousand* times higher under the scaled-*t* distribution.

VI- CONCLUSIONS

The evidence against the assumption that stock returns are normally distributed has been mounting for over thirty years. Most of the empirical evidence analyzes US data, although some recent studies have considered European markets. In this article, we turned our attention to the four Scandinavian markets, which had so far received little attention.

We started by describing the data and testing the hypothesis that stock returns in the Scandinavian markets are normally distributed. Our data shows the typical fat tails and high peaks observed in many other markets, as well as skewness in different directions; hence, the outright rejection of the normality assumption came as no surprise. These results are fully consistent with those found for many other markets and reported in other studies.

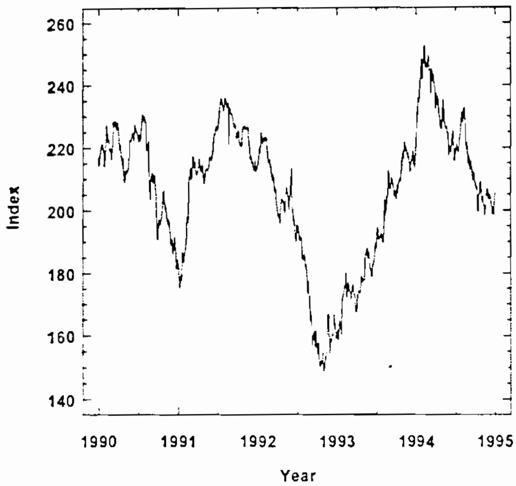
We then fitted the Normal distribution to the data, as well as four alternative specifications, all of which exhibit fat tails and one that also allows for skewness. Predictably, the Normal distribution exhibits the worst fit in all markets. The scaled-*t* distribution, on the other hand, exhibits the overall best fit and cannot be rejected at any reasonable significance level in any market.

Finally, we attempted to quantify the error that can be made by predicting unconditional stock returns by using the Normal distribution instead of the more appropriate scaled-*t* distribution. We have shown that such errors can be very large, particularly in the tails, and that the Normal distribution consistently underestimates the probability of (positive or negative) large returns. Therefore, the empirical evidence from Scandinavian securities markets points in the same direction as that from many other markets; that is, booms and crashes are much more likely to occur than what a Normal distribution would predict.

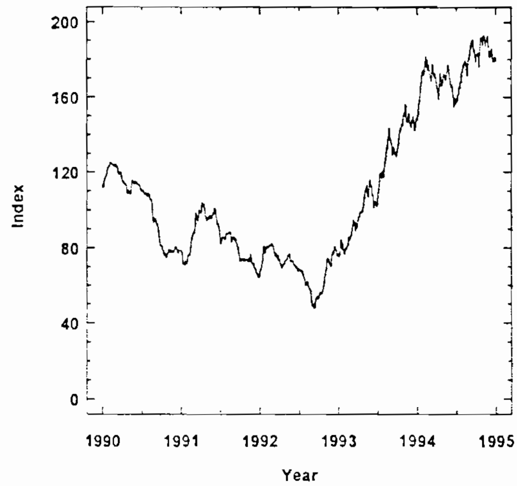
APPENDIX

A1- MARKET BEHAVIOR

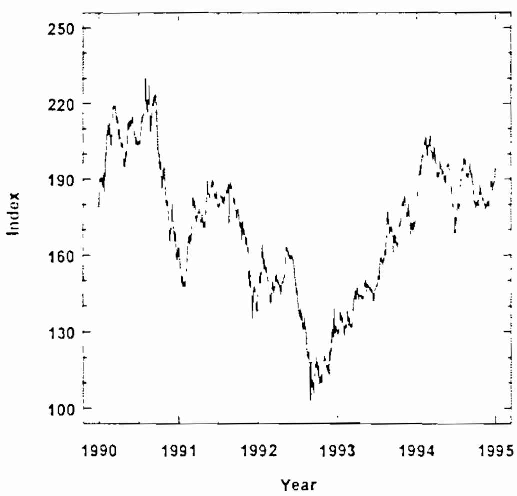
DENMARK



FINLAND



NORWAY



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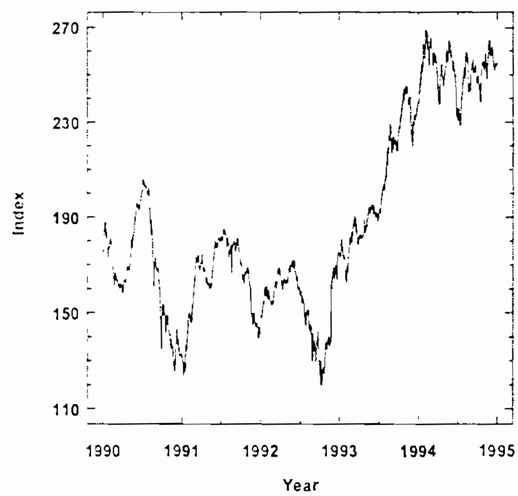

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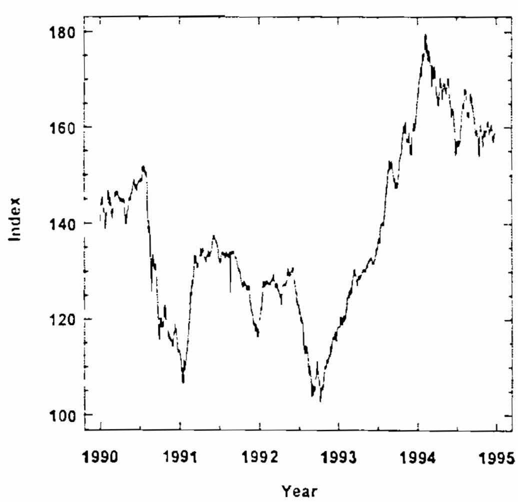

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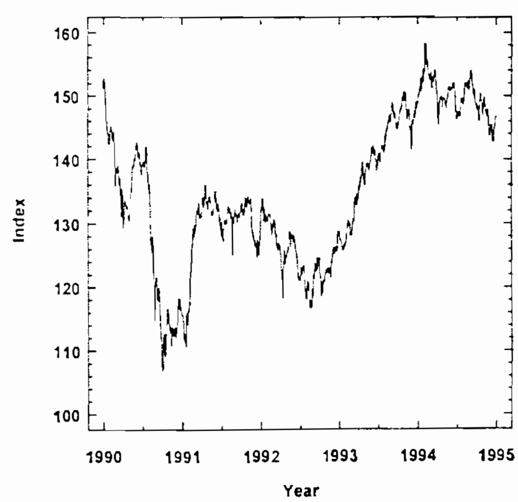
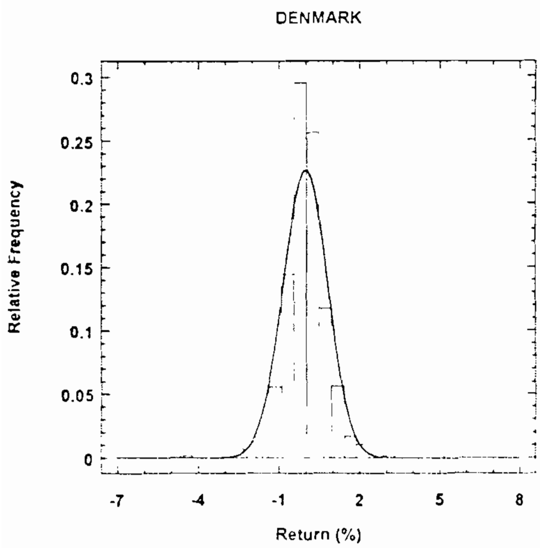
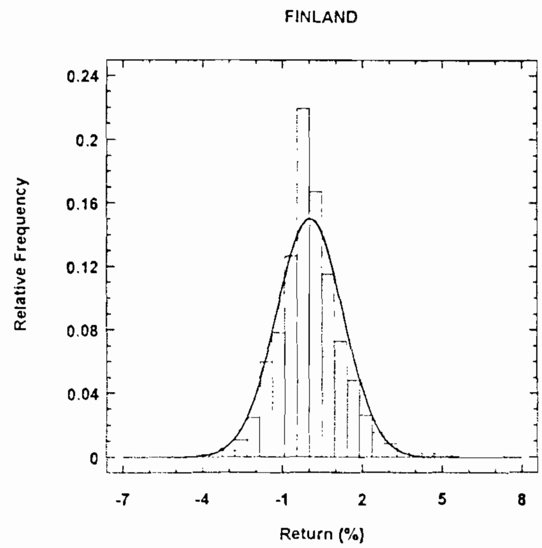
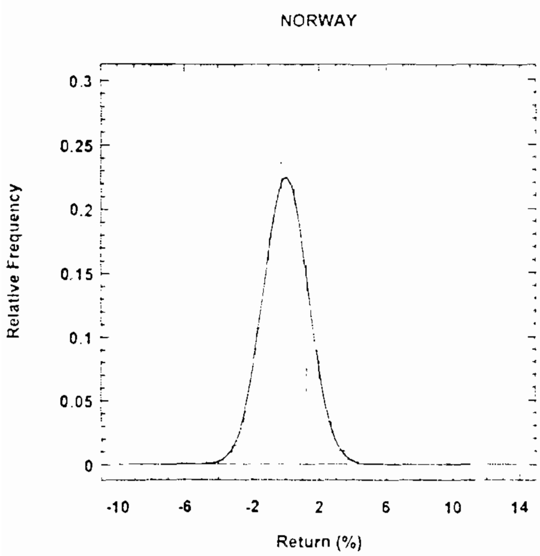
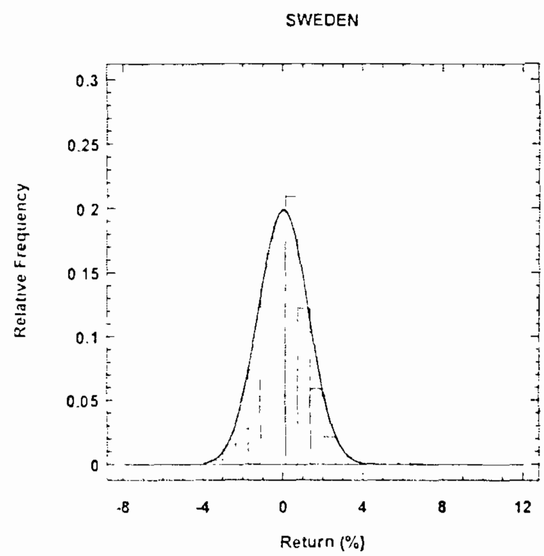


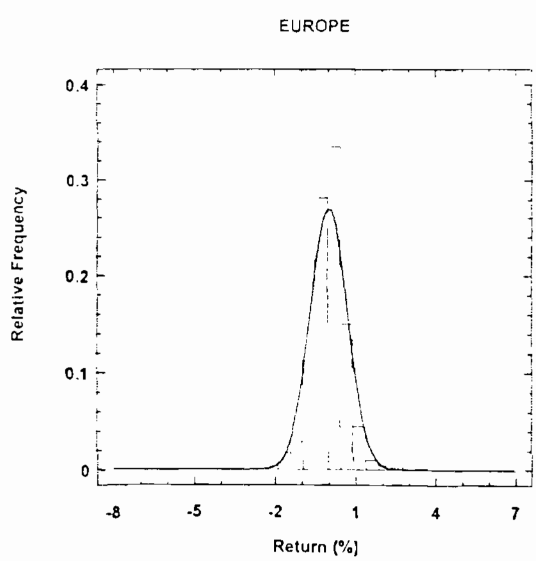
A2- HISTOGRAMS

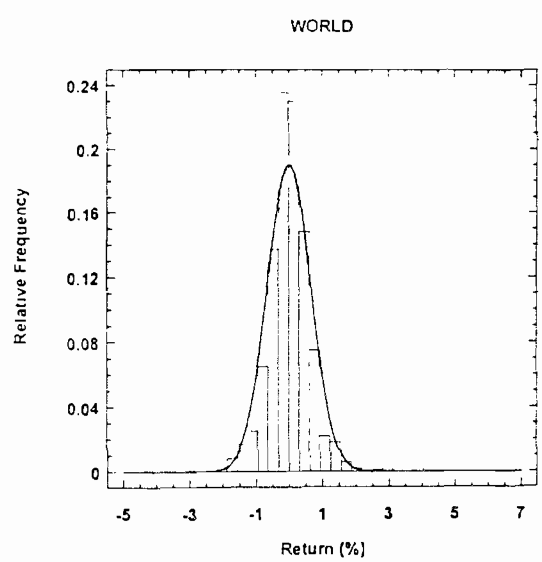












REFERENCES

Bollerslev, T., R. Chou, and K. Kroner (1992). "ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence." Journal of Econometrics, 52, 5-59.

Box, G. and G. Tiao (1973). *Bayesian Inference in Statistical Analysis.* Addison-Wesley.

Clark, P. (1973). "A Subordinated Stochastic Process Model With Finite Variance for Speculative Prices." Econometrica, 41, 135-155.

Fama, E. (1965). "The Behavior of Stock Market Prices." Journal of Business, 38, 34-105.

Gray, B. and D. French (1990). "Empirical Comparisons of Distributional Models for Stock Index Returns." Journal of Business, Finance & Accounting, 17, 451-459.

Hsu, D. (1982). "A Bayesian Robust Detection of Shift in the Risk Structure of Stock Market Returns." Journal of the American Statistical Association, March 1982, 29-39.

Johnson, N. and S. Kotz (1970). *Distributions in Statistics: Continuous Univariate Distributions.* John Wiley and Sons.

Mandelbrot, B. (1963). "The Variation of Certain Speculative Prices." Journal of Business, 36, 394-419.

Peiró, A. (1994). "The Distribution of Stock Returns: International Evidence." Applied Financial Economics, 4, 431-439.

Peters, E. (1991). *Chaos and Order in the Capital Markets. A New View of Cycles, Prices, and Market Volatility.* John Wiley and Sons.

Praetz, P. (1972). "The Distribution of Share Price Changes." Journal of Business, 45, 49-55.

Press, J. (1967). "A Compound Events Model for Security Prices." Journal of Business, 40, 317-335.

Smith, J. (1981). "The Probability Distribution of Market Returns: A Logistic Hypothesis." Ph.D. dissertation, University of Utah.

Upton, D. and S. Shanon (1979). "The Stable Paretian Distribution, Subordinated Stochastic Processes, and Asymptotic Lognormality: An Empirical Investigation." Journal of Finance, 34, 1031-1039.